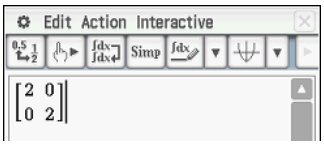
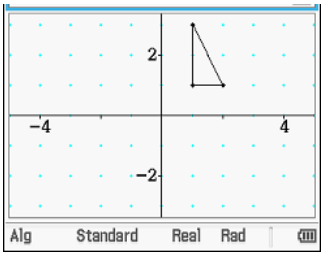
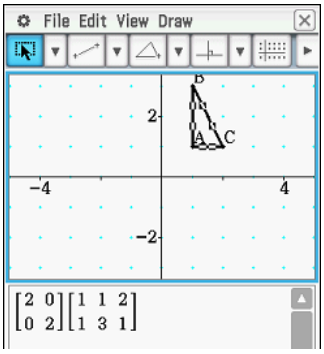
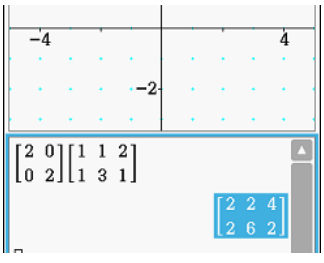
Activity 31 Visualising linear transformations

Aim: Calculate matrix products to draw images and interpret linear transformations.

1. What is the effect of applying the linear transformation $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ to a triangle with vertices at (1, 1) (1, 3) and (2, 1)?

Enter the matrix T • Open Main • Press Keyboard • Tap Math2 tap ▣ • Enter the transformation matrix	
Open a Geometry window in half the screen and draw the triangle • Tap the pull down arrow of the applications icon and tap ▣ to open a geometry window • Draw the triangle **Save the diagram** • Select [File | Save] and enter an appropriate name, e.g. triTrans	
Perform the transformation • Select the triangle • Drag into Main next to the transformation matrix • Press EXE	
Draw result • Highlight the result • Drag into the Geometry window • Select [View | Zoom to Fit]	

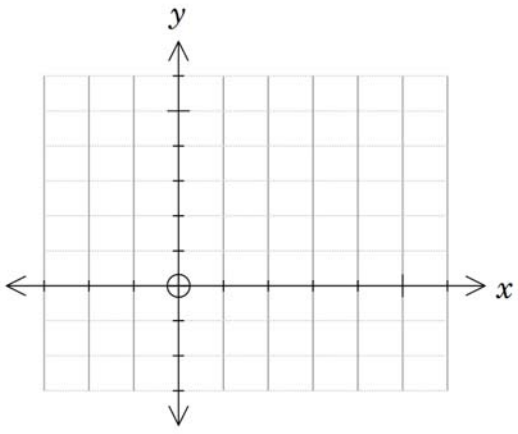
Describe the transformation.

Table of transformations you are expected to describe in this course

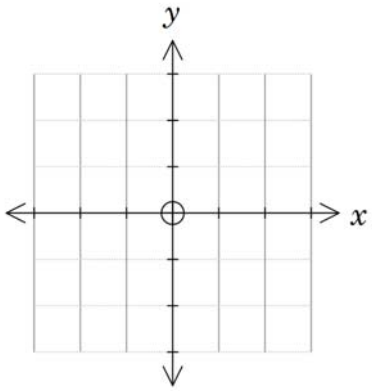
Transformation	Specified by	Example	Common terms
Rotation	Angle of rotation Centre of rotation Direction of rotation	Rotation of 90° anticlockwise about the origin	Turn Spin
Reflection	Mirror line	Reflect in the line $y = x$	Flip
Dilation	Dilation factor (and direction)	Dilate by factor 3 about the origin. Dilate by factor 2 horizontally and factor 5 vertically	Magnify Enlarge Shrink
Translation	Translation vector	Translate 3 units across and 4 units down or $\begin{bmatrix} 3 \\ -4 \end{bmatrix}$	Slide Move

2. For the following linear transformations represented by the given matrix, sketch the triangle and its image and describe the transformation(s) in full. Note: You can do this by editing the transformation matrix in Main.

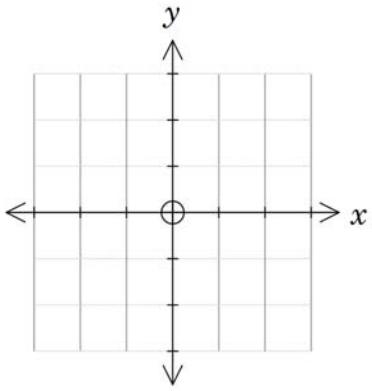
a) $\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$



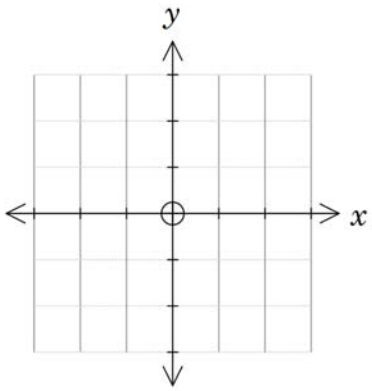
b) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$



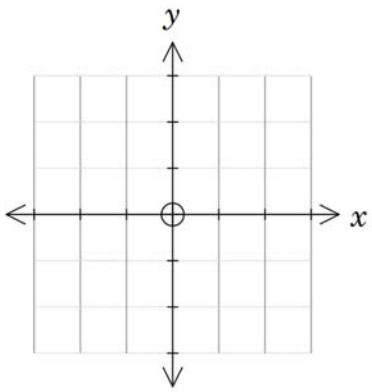
c) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$



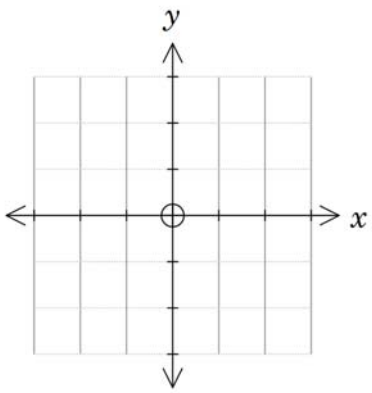
d) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$



e) $\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$



f) $\begin{bmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$



Learning notes

Q1 See the previous activity for more detail on the geometry constructions.

Q2 To do efficiently:

- Open your saved file in the Geometry window,
  Select [File | Open], choose your saved file and tap OK.
  This should just have your triangle drawn
- In Main, change the transformation matrix and press EXE.
- Drag the result into the Geometry window.

Under a linear transformation **T**, point P(x, y) moves to a point $P'(x', y')$. Points (x, y) and (x', y') are related by a pair of linear equations such as $\begin{matrix} x' = 3x - y \\ y' = x + y \end{matrix}$.

In this case, for example, $(3,2) \xrightarrow{T} (7,5)$.

We can write this as a matrix equation $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ or $\mathbf{P'} = \mathbf{TP}$ where **T** is the transformation matrix.